



Technical Report

Worst-case Dimensioning of Cluster-Tree Sensor Networks with Mobile Sink Behaviour

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Abstract

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Abstract - Modelling the fundamental performance limits of Wireless Sensor Networks (WSNs) is of paramount importance to understand the behaviour of WSN under worst-case conditions and to make the appropriate design choices. In that direction, this paper contributes with a methodology for modelling cluster-tree WSNs with a mobile sink. We propose closed-form recurrent expressions for computing the worst-case end-to-end delays, buffering and bandwidth requirements across any source-destination path in the cluster-tree. We show how to apply our theoretical results to the specific case of IEEE 802.15.4/ZigBee WSNs. Finally, we demonstrate the validity and analyse the accuracy of our methodology through a comprehensive experimental study using commercial technology, therefore validating the theoretical results through experimentation.

1. INTRODUCTION

Wireless Sensor Networks (WSNs) emerge as enabling infrastructures for large-scale distributed embedded systems. Timeliness is an important requirement to be fulfilled in these systems. However, issues such as large scale and computing/communication and energy limitations pose important difficulties in guaranteeing a correct behaviour of these systems.

Evaluating the performance limits of WSNs is therefore a crucial task, particularly when the network is expected to operate under worst-case conditions [1]. For achieving real-time communications over sensor networks, it is mandatory to rely on deterministic routing and MAC (Medium Access Control) protocols. Usually, these networks use hierarchical logical topologies such as cluster-tree or hexagonal (e.g. [2-4]). Issues such as the use of contention-free MAC protocols (e.g. time division or token passing) and the possibility of performing end-to-end resource reservation contrast with what can be achieved in mesh-like topologies, where contention-based MACs and probabilistic routing protocols are commonly used.

In a previous work [5], the authors have provided a methodology and closed form expressions to dimension the network resources in a cluster-tree WSN with a static sink. The sink – a central point that collects all sensory data – was assumed to be statically attached to the root. That work aimed at evaluating the worst-case network performance assuming a symmetric cluster-tree topology. This symmetry property was explored to derive per-hop and end-to-end resource requirements in addition to the worst-case delays of upstream flows (i.e. from child nodes to the root).

However, while the static sink behaviour is adequate for root-centric WSN applications (e.g. a surveillance system delivering alarms to a central station), other applications may impose or benefit from collecting data at different

network locations (e.g. a doctor with a hand-held computer collecting patients' status). Therefore, in this paper we investigate the worst-case resource dimensioning and analysis of cluster-tree WSNs with mobile sink behaviour.

We consider the sink as an autonomous entity that does not make part of the cluster-tree WSN, but can be associated to any of its routers through any (wired or wireless) communication means. Thus, the sink's mobility does not impact the WSN topology, but affects the data flow destination (any router in the WSN). Contrarily to [5], now not only upstream flows (sink in the root) but also downstream flows (sink not in the root) must be considered in the analysis, turning it more complete, yet more complex.

For the sake of simplicity and space, this paper does not address neither how mobility is managed (namely how routes must be updated upon mobility of the sink) nor how this procedure may impact the worst-case analysis (eventual network inaccessibility times). The paper proposes and describes a system model, an analytical methodology and a software tool that permit the worst-case dimensioning and analysis of cluster-tree WSNs. In this way, it is possible to minimize the routers' buffers size to guarantee no overflows and to minimize each cluster's duty cycle (maximizing nodes' lifetime) still satisfying that messages' deadlines are met.

Importantly, we show how it is possible to instantiate our generic methodology to be used in the design of IEEE 802.15.4/ZigBee [6, 7] systems, which are the leading technologies for wireless sensor networks. (In fact, in 2007, 7 million IEEE 802.15.4-enabled chips were sold, an increase of 1400% from 2004 and leading to roughly 50% WSN market share [7]). Finally, we assess the validity of our theoretical model, by comparing worst-case results (buffer requirements and message end-to-end delays) with the maximum and average values measured through an experimental test-bed based on Commercial Off The Shelf (COTS) technologies.

Contributions of this paper

- (1) we provide a generic system model, encompassing the cluster-tree topology model and the data flow model, we also identify the worst-case cluster scheduling for any location of the sink (Section 3).
- (2) we present a methodology, based on Network Calculus, to characterize input and output flows in each router in the cluster-tree WSN (Section 4) and to derive upper bounds on buffer requirements and per-hop and end-to-end delays (Section 5).
- (3) we show how to apply our methodology to dimension IEEE 802.15.4/ZigBee cluster-tree WSNs (Section 6).

- (4) we demonstrate the validity of our methodology through an experimental test-bed (Section 6.3).

Other related work

The evaluation of the fundamental performance limits of WSNs has addressed in several research works [1-3]. In [2], the authors have evaluated the real-time capacity of multi-hop WSNs, identifying how much real-time data the network can transfer by their deadlines. A capacity bound has been derived for (ideal) MAC protocols with fixed priority packet scheduling mechanisms. In [3], the authors have analysed the fundamental limits for acceptable loads, utilization, and delays in multi-hop sensor networks with linear and grid topologies, in case of all sensor nodes contribute equally to the network load. In [1], the authors evaluated the asymptotic behaviour of operational lifetime and energy-constrained capacity of sensor networks.

The worst-case analysis and resource dimensioning of WSNs using Network Calculus has been pursued by Schmitt *et al.* ([10-12]), who proposed the Sensor Network Calculus methodology. In [10], Sensor Network Calculus was introduced and basic components such as arrival and service curves were defined. The system model assumes generic tree-based topologies with nodes transmitting sensor data towards the sink that is associated with the root. The authors have also proposed a general iterative procedure to compute the network internal flows and, subsequently, resource requirements and delay bounds. On the contrary, our work provides recurrent equations so that to avoid iterative computations that are complex and time consuming and not suitable for large-scale WSNs. In [11], the previous Sensor Network Calculus framework was extended to incorporate in-network processing features (e.g. data aggregation) to reduce the amount of data that has to be transmitted. In our work, we abstract from the computational resources in the network nodes and from data aggregation. In [12], the authors searched for the worst-case topology (i.e. the topology that exhibits the worst-case behaviour in terms of buffer requirements, delay bounds and network lifetime) in networks with random nodes deployment. Finding the general worst-case topology is a complex task, thus their methodology explores the worst-case tree constrained on maximum depth and number of child routers that maximizes the arrival curve of the root. As compared to [10-12], our system model is more accurate for the specific case of cluster-tree topologies and the sink can be associated with any router in WSN.

On the other hand, several research works have dealt with sink mobility in order to minimize energy consumption in the network [13, 14]. The proposed approaches use random, predictable or controlled mobility of one or more sinks [13]. Four strategies (random, geographically, intelligent and genetic algorithm-based strategies) focusing on optimal sink placement for minimizing the worst-case delay as well as maximizing the lifetime of a WSN have been introduced in [14]. Conversely, in our work we compute the worst-case delays and resource requirements for given sink position.

2. BACKGROUND ON NETWORK CALCULUS

Network Calculus [9] is a mathematical methodology based on min-plus algebra that applies to the deterministic analysis of queuing/flows in communication networks. It is

concerned with worst-case rather than average-case behaviour, and may be applied to derivate deterministic guarantees on network resources. This section briefly introduces the aspects of the Network Calculus formalism that are most significant to this paper. For additional details please refer to [9].

A basic system model S in Network Calculus consists of a buffered FIFO node with the corresponding transmission link (Figure 1). For a given data flow, the *input function* $R(t)$ is a cumulative number of bits that have arrived to system S in the time interval $(0, t)$. The *output function* $R^*(t)$ is the number of bits that have left S in the same interval $(0, t)$. Both functions are wide-sense increasing, i.e. $R(t_1) \leq R(t_2)$ for all $t_1 \leq t_2$.

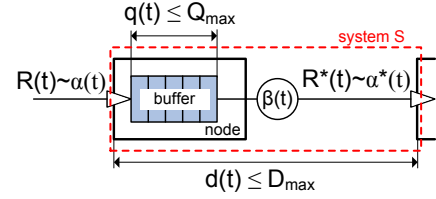


Figure 1. The basic system model in Network Calculus.

Definition 1: Arrival Curve $\alpha(t)$ (Figure 2). Let $\alpha(t)$ be a wide-sense increasing function for $t \geq 0$. Then an incoming flow with input function R is upper bounded by α iff for $\forall s, 0 \leq s \leq t, R(t) - R(s) \leq \alpha(t - s)$. It is also said that R is α -smooth or R is constrained by α , i.e. $R(t) \sim \alpha(t)$.

Definition 2: Service Curve $\beta(t)$ (Figure 2). Consider system S and a flow through S with input and output functions R and R^* respectively. Then S offers to the traversing flow a service curve β iff β is wide-sense increasing function, with $\beta(0) = 0$, and for $\forall t$ exists $t_0 \leq t$ such that $R^*(t) - R^*(t_0) \geq \beta(t - t_0)$. This means that an outgoing flow with output function R^* during any period $(t - t_0)$ is at least equal to $\beta(t - t_0)$.

In brief, an arrival curve constraints the incoming data flow into a node, and a service curve keeps the characteristics with which cumulated flow is forwarded by the node to the following node in the direction of the sink.

Now, the knowledge of the arrival and service curves enables us to provide the performance bounds for a lossless system, namely the delay bound D_{max} , which represents the worst-case delay of the message traversing the system S , and the backlog bound Q_{max} , which represents the maximum queue length of the flow, i.e. the minimum buffer size requirement inside the system S .

Theorem 1: Delay Bound D_{max} . Let a flow with input function $R(t)$, constrained by arrival curve $\alpha(t)$, traverses a system S that offers a service curve $\beta(t)$. Then the delay bound is the maximum horizontal distance between $\alpha(t)$ and $\beta(t)$ and for all $t \geq 0$, the delay $d(t)$ satisfies:

$$d(t) \leq \sup_{s \geq 0} \{ \inf \{ \tau \geq 0 : \alpha(s) \leq \beta(s + \tau) \} \} = D_{max} \quad (1)$$

Theorem 2: Backlog Bound Q_{max} . Let a flow with input function $R(t)$, constrained by arrival curve $\alpha(t)$, traverses a system S that offers a service curve $\beta(t)$. Then the backlog bound is the maximum vertical distance between $\alpha(t)$ and $\beta(t)$ and for all $t \geq 0$, the backlog $q(t)$ satisfies:

$$q(t) \leq \sup_{s \geq 0} \{ \alpha(s) - \beta(s) \} = Q_{max} \quad (2)$$

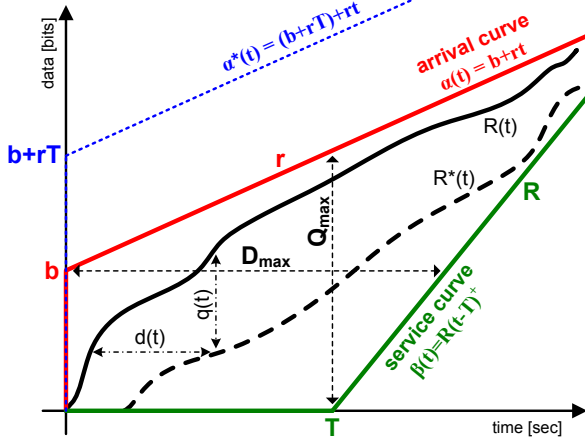


Figure 2. Example of a cumulative input function $R(t)$ constrained by (b, r) arrival curve $\alpha(t)$ and a cumulative output function $R^*(t)$ constrained by rate-latency service curve $\beta(t)$.

With Network Calculus is also possible to express an upper bound of the outgoing flow with output function $R^*(t)$, called *output bound*. So far we have handled a system S as a single buffered node (Figure 1). On the other hand, the system S might be also a sequence of nodes or even a complete network, for example. Then the *concatenation theorem* enables us to investigate systems in sequence as a single system.

Definition 3: Min-Plus Convolution. Let f and g be wide-sense increasing functions and $f(0) = g(0) = 0$. Then their convolution under min-plus algebra is defined as

$$(f \otimes g)(t) = \inf_{0 \leq s \leq t} \{f(t-s) + g(s)\}; \text{ for } \forall t \geq 0 \quad (3)$$

Definition 4: Min-Plus Deconvolution. Let f and g be wide-sense increasing functions and $f(0) = g(0) = 0$. Then their deconvolution under min-plus algebra is defined as

$$(f \oslash g)(t) = \sup_{s \geq 0} \{f(t+s) - g(s)\}; \text{ for } \forall t \in \mathbb{R} \quad (4)$$

Theorem 3: Output Bound $\alpha^*(t)$. Assume a flow with input function $R(t)$, constrained by arrival curve $\alpha(t)$, traverses a system S that offers a service curve $\beta(t)$. Then the output function $R^*(t)$ is upper bounded by the following arrival curve

$$\alpha^*(t) = (\alpha \otimes \beta)(t) \geq \alpha(t) \quad (5)$$

Theorem 4: Concatenation of Nodes. Assume a flow with input function $R(t)$ traverses system S_1 and S_2 in sequence, where S_1 offers service curve $\beta_1(t)$ and S_2 offers $\beta_2(t)$. Then the concatenation of these two systems offers the following single service curve $\beta(t)$ to the traversing flow:

$$\beta(t) = (\beta_1 \otimes \beta_2)(t) \quad (6)$$

Due to the accumulation of the data flows in the direction of the sink, the nodes offer a service curve $\beta(t)$ to this aggregated data flow. Using the aforementioned expressions (1) and (2), the delay and backlog bounds can be computed for entire aggregate data flow at each node. Using the *aggregate scheduling* theorem, the tighter bounds can be computed for individual flows traversing the network. In this paper, we use both approaches (i.e. entire aggregate flow per node or individual flow over network) to compare the results.

Theorem 5: Aggregate Scheduling. Consider a node multiplexing two data flows, 1 and 2, in FIFO order. Assume that flow 2 is constrained by the arrival curve $\alpha_2(t)$ and the node guarantees a service curve $\beta(t)$ to the aggregate of these two flows. Define the family of functions as

$$\beta_1(t, \theta) = (\beta(t) - \alpha_2(t - \theta))^+ \cdot 1_{\{t > \theta\}} \quad (7)$$

Then for any $\theta \geq 0$, $\beta_1(t, \theta)$ is a service curve guaranteed for flow 1.

So far we have considered abstract network calculus model. The accuracy of the worst-case bounds depends on how tight the selected arrival and service curves follow the real network behaviour. Different types of arrival and service curves have been proposed in Network Calculus (e.g., [9, 10]). However, the (b, r) arrival curve and *rate-latency service curve* are the most used in such network models. These curves lead to a fair trade-off between computing complexity and approximation accuracy of the real system behaviour, as it will be seen throughout the rest of the paper.

The affine or (b, r) arrival curve (Figure 2) is defined as $\alpha(t) = b + r \cdot t$ for $\forall t > 0$ and 0 otherwise, where b is called burst tolerance, which is the maximum number of bits that can arrive simultaneously at a given time to the system S (in bits) and r is the average data rate (in bps). This type of arrival curve represents a data flow based on the average sensing rate with short-term fluctuations given by the burst tolerance, i.e. it allows a node to send b bits at once, but no more than an average of r bits per second over long run.

The *rate-latency service curve* is defined as $\beta_{R,T}(t) = R \cdot (t - T)^+$, where $R \geq r$ is the forwarding rate, T is the latency of forwarding data - both depend on the processing speed and resource allocation mechanism, for example, and $(x)^+ = \max(0, x)$. If $r > R$, the bounds are infinite.

Hereafter, we consider a system S that offers a rate-latency service curve $\beta_{R,T}(t)$ and that stores input data in FIFO-order buffer. Then the performance bounds D_{max} and Q_{max} (see Figure 2 for additional intuition) guaranteed to the data flow, constrained by the (b, r) arrival curve $\alpha(t)$ and traversing system S , are easily computed as:

$$D_{max} = \frac{b}{R} + T \quad Q_{max} = b + r \cdot T \quad (8)$$

An application of (5) to a data flow constrained by (b, r) arrival curve $\alpha(t)$ and traversing system S guaranteeing a rate-latency service curve $\beta_{R,T}(t)$, the output bound of this data flow is expressed as (the proof can be found in [16])

$$\alpha^*(t) = \alpha(t) \otimes \beta_{R,T}(t) = \alpha(t) + r \cdot T \quad (9)$$

According to the aggregate scheduling theorem we assume that $\alpha_2(t)$ is a (b, r) arrival curve and $\beta(t)$ is a rate-latency service curve $\beta_{R,T}(t)$, then an equivalent service curve for data flow 1 (7) is expressed as

$$\beta_1(t, \theta) = (R - r_2) \left[t - \left(\frac{b_2 + r_2(T - \theta)}{R - r_2} + T \right) \right]^+ \cdot 1_{\{t > \theta\}} \quad (10)$$

3. SYSTEM MODEL

This section defines the cluster-tree topology and data flow models that will be considered in the analysis. It also

elaborates on the worst-case cluster scheduling; that is, the time sequence of clusters' active periods leading to the worst-case end-to-end delay for a message to be routed to the sink.

3.1 Cluster-Tree Topology Model

Cluster-tree WSNs feature a tree-based logical topology, where nodes are organized in different groups, called *clusters*. Each node is connected with a maximum of one node at lower depth, called *parent node*, and can be connected with multiple nodes at upper depth, called *child nodes*. This topology needs also corresponding routing algorithm to realize the data transmission through multiple levels. The transmission using multiple hops is easy and fast because each node can interact only with its pre-defined parent and child nodes.

Consider Figure 3. The cluster-tree topology contains two main types of nodes. First, the nodes that can associate with previously associated nodes and can participate in the multi-hop routing are referred to as *routers* (R_{ij} , i.e. the j^{th} router at depth i). Second, the leaf nodes that do not allow association of other nodes and do not participate in routing are referred to as *end-nodes* (N). The router that has no parent is called *root* (it might hold special functions such as identification, formation and control of the entire topology). Routers and end-nodes can both have sensing capabilities. Therefore they are generally referred to as *sensor nodes*. Each router forms its cluster and is referred to as *cluster-head* of this cluster. All child nodes (i.e. end-nodes and routers) of given cluster-head are associated to its cluster, and the cluster-head handles all of their data transmissions. It results that each router (except the root) is a member in two clusters, once as a child and once as a parent (i.e. a cluster-head). Hence, the data communication in cluster-tree topology can be considered to be cluster-oriented.

In this paper we aim at specifying the worst-case cluster-tree topology, i.e. the network topology configuration that leads to the worst-case performance. This means that a dynamically changing cluster-tree WSN can assume different configurations, but it can never exceed the worst-case topology, in terms of maximum depth and number of child routers/end-nodes. Thus, the worst-case cluster-tree topology is graphically represented by a rooted balanced directed tree [15] defined by the following three parameters:

- H : Height of the tree, i.e. the maximum number of logical hops for a message from the deepest router to reach the root (including the root as a final hop). A tree with only a root has a height of zero.
- $N_{\text{end_node}}^{\text{MAX}}$: Maximum number of end-nodes that can be associated to a router and have been allocated resource guarantees (e.g. time slots or bandwidth).
- $N_{\text{router}}^{\text{MAX}}$: Maximum number of child routers that can be associated to a parent router and have been allocated resource guarantees (e.g. time slots or bandwidth).

The *depth* of a node is defined as the number of logical hops from that node to the root. The root is at depth zero, and the maximum depth of an end-node is $H+1$.

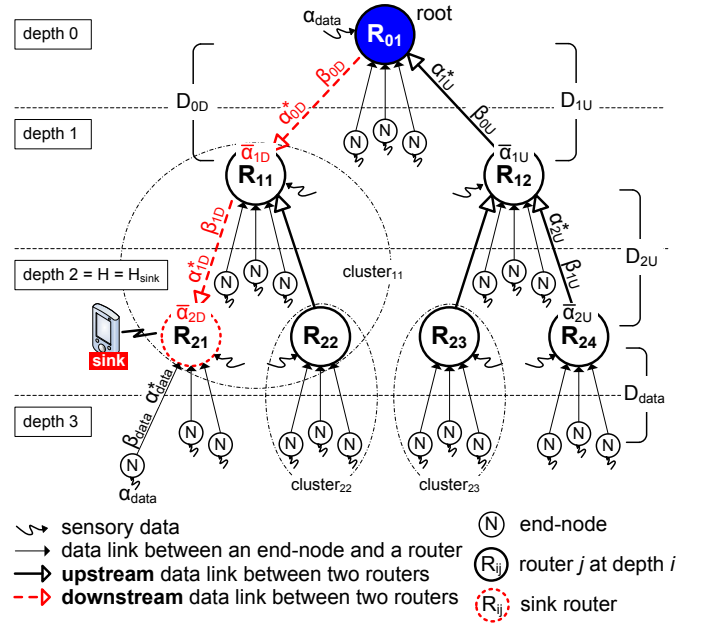


Figure 3. The cluster-tree topology and data flow models.

Note that the *sink* is a special type of node that gathers the sensory data from all sensor nodes inside the network. Unlike previous work, we relax the assumption that the sink is only associated with the root and consider the sink to be an autonomous and topology-independent mobile node. The mobile behaviour means that a sink moves arbitrarily within a cluster-tree WSN and can be associated with any router within communication range. The router, to which the sink is in a given moment associated, is referred to as *sink router*. There can be more than one mobile sink in a WSN, but we assume that only one is active (i.e. gathers the sensory data) at a given time. We specify another parameter, $H_{\text{sink}} \in (0, H)$, to represent the depth at a given moment of the sink router in a cluster-tree topology. Note that if the sink is associated with the root, i.e. $H_{\text{sink}} = 0$, the network contains only upstream flows. This simpler case has already been analysed in [5]. In this paper, we analyze the case where $H_{\text{sink}} > 0$.

Our terminology and conventions are as illustrated in Figure 3, corresponding to a configuration where $H = 2$, $N_{\text{end_node}}^{\text{MAX}} = 3$, $N_{\text{router}}^{\text{MAX}} = 2$, and $H_{\text{sink}} = 2$. Note that a cluster-tree WSN may contain additional nodes per router than those defined by $N_{\text{router}}^{\text{MAX}}$ and $N_{\text{end_node}}^{\text{MAX}}$ parameters. However, these additional nodes cannot be granted guaranteed resources.

3.2 Data Flow Model

In this paper, we assume that all sensory data is exclusively sent to the sink. All sensor nodes are assumed to sense and transmit data upper bounded by the arrival curve $\alpha_{\text{data}}(t) = b_{\text{data}} + r_{\text{data}} \cdot t$, where b_{data} is burst tolerance and r_{data} is average data rate. In case of different data flows, $\alpha_{\text{data}}(t)$ is considered to represent the upper bound of the highest flow in a network. This may introduce some pessimism to the analysis if the variance between data flows is significant.

Each end-node is granted a service guarantee from its parent router corresponding to the rate-latency service curve $\beta_{\text{data}}(t) = R_{\text{data}} \cdot (t - T_{\text{data}})^+$, where $R_{\text{data}} \geq r_{\text{data}}$ is the guaranteed link bandwidth and T_{data} is the maximum latency of the service. The same service curve is provided

to all end-nodes by their parent routers. By applying Eq. (9) to a flow constrained by the arrival curve $\alpha_{data}(t)$ and that is granted a service curve $\beta_{data}(t)$, we obtain the output arrival curve $\alpha_{data}^*(t)$, which upper bounds the outgoing data flow from any end-node:

$$\alpha_{data}^*(t) = \alpha_{data}(t) + r_{data} \cdot T_{data} \quad (11)$$

On the other hand, the amount of bandwidth allocated by each router depends on the cumulative amount of data at its inputs, which increases towards the sink. Thus, the total input function $R(t)$ of each router depends on the depth, and consists of the sum of the output functions $R^*(t)$ of its end-nodes and child routers. Additionally, the router itself can be equipped with sensing capability producing a traffic bounded by $\alpha_{data}(t)$. Thus, the arrival curve constraining the total input function $R(t)$ of a router at general depth i is expressed as:

$$\bar{\alpha}_i = \alpha_{data} + N_{end_node}^{MAX} \cdot \alpha_{data}^* + \sum_{j=1}^{N_{router}^{MAX}} \alpha_{router(i+1,j)}^* \quad (12)$$

This result can then be used in Eq. (9). The outgoing flow of a router at depth i , that guarantees service curve β_{i-1} , is upper bounded by the output arrival curve as follows:

$$\alpha_i^* = \bar{\alpha}_i \odot \beta_{i-1} \quad (13)$$

Hence, the data flow analysis consists in the computation of the arrival curves $\bar{\alpha}_i$ and α_i^* , using iteratively Eqs. (12) and (13), from the deepest routers until reaching the sink. After that, the resource requirements of each router, in terms of buffer requirement Q_i and bandwidth requirement R_i , and the worst-case end-to-end delay bound of WSN are computed.

In cluster-tree WSNs where the sink can be associated with a router other than the root, data flows may then be redirected in the downstream directions. Data flows over upstream links (called *upstream flows*) have already been analysed in [5]. Data flows over downstream links (called *downstream flows*), where data is sent from a parent router to its child router, are analysed in this paper. Note that in the downstream case, the parent router must reserve enough bandwidth for its own outgoing data, in contrast to the upstream case, where a parent router must reserve enough bandwidth for the outgoing data of its child nodes. In what follows, the upstream and downstream flows are marked by the subscripts **U** and **D**, respectively (e.g. α_{iu}^* , α_{id}^*). We also assume two types of service curves (i.e. β_{iu} for upstream flows and β_{id} downstream flows) provided by each parent router at depth i to its child routers at depth $i+1$, and expressed as:

$$\beta_{iu}(t) = R_{iu} \cdot (t - T_{iu})^+ \quad \beta_{id}(t) = R_{id} \cdot (t - T_{id})^+ \quad (14)$$

, where R_i is the guaranteed link bandwidth, which must be greater than or equal to the rate of outgoing data flow, and T_i is maximum latency that a data must wait for a service. To ensure the symmetry properties of the worst-case cluster-tree topology assumed in our methodology, the same downstream or upstream service curves must be guaranteed to all downstream or upstream flows at a given depth, respectively. Note that, the routers forwarding data flows in the upstream direction are referred to as *upstream routers* (e.g. R_{12} or R_{23} in the example in Figure 3), whereas the routers forwarding the downstream flows are

referred to as *downstream routers* (e.g. R_{01} or R_{11}). In the same way, the wireless links are referred to as upstream or downstream.

3.3 Time Division Cluster Scheduling

In general, the radio channel is a shared communication medium where more than one node can transmit at the same time. In cluster-tree WSNs, messages are forwarded from cluster to cluster until reaching the sink. The time period of each cluster is periodically divided into an *active period* (AP), during which the cluster-head enables data transmissions inside its cluster, and a subsequent *inactive period*, during which all cluster nodes may enter low-power mode to save energy resources. Note that a router must be awake during its active period and active period of its parent router. To avoid collisions between multiple clusters, it is mandatory to schedule active periods of different clusters in an ordered sequence, called *Time Division Cluster Schedule* (TDCS). In other words, TDCS is equivalent to a permutation of active periods of all clusters in a WSN such that no inter-cluster interference occurs. In case of one collision domain (i.e. all nodes hear each other), the TDCS must be non-overlapping, i.e. only one cluster can be active at any time. Hence, the length of TDCS is given by the number of clusters and the duration of their active periods. On the contrary, in a network with multiple collision domains, the clusters from different non-overlapping collision domains may be active at the same time.

Due to the cumulative flow effect, the amount of traffic increases in the direction of the sink such that the maximum flow is reached in the cluster to which the sink is associated (e.g. *cluster₁₁* in Figure 3). Hence, the duty cycles of the clusters closer to the sink should be higher (i.e. longer APs) than the further ones, to ensure efficient bandwidth utilization [17]. Note that the ratio of active to inactive period is called duty cycle.

The TDCS significantly affects the resource requirements and delay bounds in cluster-tree WSNs. The number of feasible TDCSs in a network with n routers inside one collision domain is equal to the number of permutations given by n factorial ($n!$). Note that for each data flow originated in a given node, there is a corresponding best-case/worst-case TDCS that minimizes/maximizes the end-to-end delay of that flow, respectively. Thus, it is impossible to determine a general best-case or worst-case TDCS meeting the requirements of all data flows. On one hand, the best-case TDCS of a data flow originated in node r comprises the consecutive sequence of active periods corresponding to the ordered sequence of the clusters traversed along the routing path from r to the sink. On the other hand, the worst-case TDCS comprises the same ordered sequence of active periods, but in the reverse order, which means starting from the sink backward to the node r . The active periods of other clusters, which are not on the routing path, are appended to the previously formed sequence in arbitrary order such that a complete TDCS is produced. Using our methodology based on the symmetric properties of cluster-tree topology, the network resources of a WSN are dimensioned for the worst-case TDCS of a data flow originated in the end-node that is farthest from the sink (i.e. a flow along the longest path in a WSN).

Let us consider example in Figure 3, where an end node of router R_{24} sends sensory data to the sink associated with the router R_{21} , i.e. a flow along the longest path in WSN. The best-case TDCS of aforementioned flow comprises the continuous sequence of active periods in the following order: $AP_{24}, AP_{12}, AP_{01}, AP_{11}$, where AP_i is the active period of $cluster_i$. On the contrary, the worst-case TDCS comprises the same sequence in reverse order: $AP_{11}, AP_{01}, AP_{12}, AP_{24}$. The active periods of other clusters are appended in arbitrary order such that the complete worst-case TDCS is the sequence: $AP_{21}, AP_{11}, AP_{01}, AP_{12}, AP_{24}, AP_{22}, AP_{23}$, for example.

To reduce the resource requirements of the routers, we introduce the following *priority rule*: “When a router handles the links in different directions (e.g. R_{01} in Figure 3), the incoming flows via upstream data links are served before the outgoing flow via downstream data link.” Using this rule, the end-to-end delay of an incoming data flow can be reduced to at most one TDCS cycle duration.

4. DATA FLOWS ANALYSIS

In this section, we derive recurrent equations of the input and output downstream flows as a function of the router's depth considering the cluster-tree topology model for WSN presented in Section 3.

In our model, we assume that the end-nodes have sensing capabilities, but the sensing capability of routers is optional. For an improved analysis, we introduce a binary variable S whose value is equal to 1 if routers have sensing capabilities; otherwise S is equal to 0.

The total input data flow of each router as shown in Eq. (12) comprises, among other terms, the sum of the output flows of its end-nodes and, optionally, its own sensory data flow constrained by $\alpha_{data}(t)$. This part of the total input flow is the same for upstream and downstream flows, hence we introduce the substitution:

$$\bar{\alpha}_H(t) = S \cdot \alpha_{data}(t) + N_{end_node}^{MAX} \cdot \alpha_{data}^*(t)$$

Thus, using Eq. (11) we get:

$$\bar{\alpha}_H(t) = (N_{end_node}^{MAX} + S) \cdot \alpha_{data}(t) + N_{end_node}^{MAX} \cdot r_{data} \cdot T_{data} \quad (15)$$

where, $\bar{r}_H = (N_{end_node}^{MAX} + S) \cdot r_{data}$ is the resulting aggregate rate of $(N_{end_node}^{MAX} + S)$ input data flows, and $\bar{b}_H = (N_{end_node}^{MAX} + S) \cdot b_{data} + N_{end_node}^{MAX} \cdot r_{data} \cdot T_{data}$ is the burst tolerance. Note that $\bar{\alpha}_H(t)$ is also equal to the total input upstream flow of the deepest routers (at depth H).

4.1 Upstream Data Flows

In [5], the output and input upstream flows were analysed and derived in detail. Thus, here we only summarize the final general recurrent expressions. The arrival curve, constraining the total input upstream flow of each router at depth i , is expressed as follows:

$$\alpha_{iU} = \left(\sum_{j=0}^{H-i} (N_{router}^{MAX})^j \right) \cdot \bar{\alpha}_H + \sum_{j=1}^{H-i} ((N_{router}^{MAX})^j \cdot \sigma_{i+j-1}) \quad (16)$$

$$\text{for } \forall i, 0 \leq i \leq H, \quad \text{where } \sigma_n = \left(\sum_{k=0}^{H-(n+1)} (N_{router}^{MAX})^k \right) \cdot \bar{r}_H \cdot T_{nU}$$

The output bound for the upstream data flow from each child router at depth i , receiving a service curve $\beta_{i-1}(t)$ from a parent router at depth $i-1$, is then expressed as:

$$\alpha_{iU}^* = \bar{\alpha}_{iU} + \sigma_{i-1} \\ = \left(\sum_{j=0}^{H-i} (N_{router}^{MAX})^j \right) \cdot \bar{\alpha}_H + \sum_{j=0}^{H-i} ((N_{router}^{MAX})^j \cdot \sigma_{i+j-1}) \quad (17)$$

for $\forall i, 0 < i \leq H$

4.2 Downstream Data Flows

We evaluate the arrival curve of the total input downstream flow $\bar{\alpha}_{iD}$ and the upper bound of the output downstream flow α_{iD}^* depth by depth, using the Network Calculus methodology, starting from depth 0 (i.e. the root). In our analysis, we consider the queuing model in Figure 4.

Analysis of depth 0

At depth 0, there is only one router, the root, and its total input data flow comprises the sum of the output flows of its end-nodes, the sum of the output upstream flows of its $(N_{router}^{MAX} - 1)$ child routers, and, optionally, its own sensory data flow constrained by $\alpha_{data}(t)$. Thus, the arrival curve constraining the total input data flow is expressed as:

$$\bar{\alpha}_{0D} = \bar{\alpha}_H + (N_{router}^{MAX} - 1) \cdot \alpha_{1U}^*$$

As a result, applying Eq. (17) we obtain:

$$\bar{\alpha}_{0D} = (N_{router}^{MAX})^H \cdot \bar{\alpha}_H + (N_{router}^{MAX} - 1) \cdot \delta_0 \quad (18)$$

$$\text{where } \delta_0 = \sum_{j=0}^{H-1} ((N_{router}^{MAX})^j \cdot \sigma_j)$$

The total input flow $\bar{\alpha}_{0D}$ is forwarded by the root to one of its child routers in the sub-tree where the sink is associated. The root provides a service curve $\beta_{0D}(t) = R_{0D} \cdot (t - T_{0D})^+$. According to Eq. (9), the output flow from the root at depth 0 to a child router at depth 1 is then upper bounded by the curve:

$$\alpha_{0D}^* = \bar{\alpha}_{0D} \odot \beta_{0D} = \bar{\alpha}_{0D} + \tau_0 \quad (19)$$

$$\text{where } \tau_0 = (N_{router}^{MAX})^H \cdot \bar{r}_H \cdot T_{0D}$$

Analysis of depth 1

The total input downstream flow of a router at depth 1 comprises the output downstream flow of its parent router (i.e. the root, at depth 0) in addition to the flow of its child end-nodes/routers, and its own (optional) traffic. Thus, the arrival curve constraining the total input data flow is expressed as:

$$\bar{\alpha}_{1D} = \bar{\alpha}_H + (N_{router}^{MAX} - 1) \cdot \alpha_{2U}^* + \alpha_{0D}^*$$

As a result, applying Eqs. (17) and (19) we obtain:

$$\bar{\alpha}_{1D} = \left(((N_{router}^{MAX})^H + (N_{router}^{MAX})^{H-1}) \cdot \bar{\alpha}_H + (N_{router}^{MAX} - 1) \cdot (\delta_0 + \delta_1) + \tau_0 \right) \quad (20)$$

$$\text{where } \delta_1 = \sum_{j=0}^{H-2} ((N_{router}^{MAX})^j \cdot \sigma_{j+1})$$

5.1 Per-Router Resources Analysis

We aim at specifying the minimum bandwidth of each downstream data links and the minimum buffer size at each downstream router needed to store the bulk of data incoming through the router's inputs.

5.1.1 Bandwidth Requirements

Consider a parent router at depth i providing a service curve $\beta_{iD}(t)$ to its child router at depth $i+1$ (see Figure 3). The total input downstream flow of the parent router is constrained by the arrival curve $\bar{\alpha}_{iD}(t)$ and dispatched through a downstream link to its child router. Thus, to ensure a bounded delay, the guaranteed amount of bandwidth R_{iD} must be greater than or equal to the arrival rate of total input downstream flow \bar{r}_{iD} . As a result, by applying Eqs. (24) and (15) we obtain:

$$R_{iD} \geq \bar{r}_{iD} = r_{iD}^* = \left(\sum_{j=0}^i (N_{router}^{MAX})^{H-j} \right) \cdot \bar{r}_H$$

$$= \left(\sum_{j=0}^i (N_{router}^{MAX})^{H-j} \right) \cdot (N_{end_node}^{MAX} + S) \cdot r_{data} \quad (28)$$

for $\forall i, 0 \leq i < H_{sink}$.

Note that it is possible to determine the *total number of routers* in a network using Eq. (28) by having $i = H$ and $\bar{r}_H = 1$, which is expressed as:

$$\Sigma(N_{router}^{MAX}, H) = \sum_{j=0}^H (N_{router}^{MAX})^{H-j} \quad (29)$$

5.1.2 Buffer Requirements

At each router the incoming data must be stored in a buffer before its dispatching. To avoid buffer overflow, the buffer of a downstream router at depth i must be able to store all incoming data, constrained by the arrival curve $\bar{\alpha}_{iD}(t)$, until it is dispatched through the downstream link to a child router at depth $i+1$. The required buffer size Q_{iD} of the downstream router at depth i must be at least equal to the burst tolerance b_{iD}^* of the output bound $\alpha_{iD}(t)$ (see Figure 2). Hence, according to Eq. (25) we get:

$$Q_{iD} = b_{iD}^* = b_{iD}^{BURST} + b_{iD}^{UP_LATENCY} + b_{iD}^{DOWN_LATENCY} \quad (30)$$

$$= \left(\sum_{j=0}^i (N_{router}^{MAX})^{H-j} \right) \cdot \bar{b}_H + (N_{router}^{MAX} - 1) \cdot \sum_{j=0}^i \delta_j + \sum_{j=0}^i \tau_j$$

for $\forall i, 0 \leq i < H_{sink}$.

Observe that the buffer requirement is the sum of three terms. The first term is the sum of burst tolerances b_{data} of the sensory data flows of all sensor nodes inside all sub-trees of a given router. The second and third terms represent the cumulative effect of the service latency at each depth for upstream and downstream flows, respectively.

In case of a sink router at depth H_{sink} , the buffer requirement must be greater than or equal to the burst tolerance $\bar{b}_{(H_{sink})D}$ of total input flow $\bar{\alpha}_{(H_{sink})D}$ given by Eq. (26) or Eq. (27).

5.2 End-to-End Delay Analysis

The *worst-case end-to-end delay* is the delay bound of a data flow transmitted along the longest path in the

network. There are two approaches to compute this queuing delay.

5.2.1 Per-hop End-to-End Delay

The first approach consists in computing the per-hop delay bounds of the aggregate input flows, and then deducing the end-to-end delay bound as the sum of per-hop delays. According to Eq. (8), the delay bound between a parent router at depth i , which offers service curve $\beta_{iD}(t)$ to its total input downstream flow constrained by arrival curve $\bar{\alpha}_{iD}(t)$, and its child router at depth $i+1$ is expressed as: $D_{iD} = \bar{b}_{iD}/R_{iD} + T_{iD}$. In case of the upstream flow, the delay bound between a child router at depth i and its parent router at depth $i-1$ offering service curve $\beta_{(i-1)U}(t)$ has been derived in [5], and is expressed as: $D_{iU} = \bar{b}_{iU}/R_{(i-1)U} + T_{(i-1)U}$.

Hence, the maximum end-to-end delay is the sum of all per-hop delay bounds as follows:

$$D_{e2e}^{MAX} = D_{data} + \sum_{i=1}^H D_{iU} + \sum_{i=0}^{H_{sink}-1} D_{iD} \quad (31)$$

where $D_{data} = b_{data}/R_{data} + T_{data}$ is the delay bound between an end-node and its parent router.

This approach is a bit pessimistic, since the delay bound at each router is computed for the aggregation of input flows. Tighter end-to-end delay bounds can be computed for individual flows, as follows.

5.2.2 Per-flow End-to-End Delay

The idea of this approach is to derive the service curves offered to an individual flow F by the routers along the path, using the aggregate scheduling theorem in Eq. (10), and then deduce the network-wide service curve for flow F based on the concatenation theorem. Finally, according to Eq. (8), the end-to-end delay bound of a given flow F will be computed using the network-wide service curve applied to the arrival curve of the input flow. The maximum end-to-end delay is equal to the delay bound of a data flow along the longest path in the network. The complete algorithm has been presented in [5], and it is valid for upstream as well as for downstream flows.

6. APPLICATION TO IEEE 802.15.4/ZIGBEE

So far, we have analysed the general methodology for providing timeliness guarantees in cluster-tree WSNs with mobile sink behaviour independently of any specific protocol. In this section, we show how to apply the aforementioned methodology to the specific case of IEEE 802.15.4/ZigBee cluster-tree WSNs.

6.1 Overview of IEEE 802.15.4/ZigBee Protocols

The IEEE 802.15.4/ZigBee [6, 7] protocols have recently been adopted as a communication standard for WSNs. The IEEE 802.15.4 standard specifies the physical and MAC layers; the network and application layers are defined by ZigBee standard. The MAC layer supports the beacon-enabled or non beacon-enabled modes. We only consider the beacon-enabled mode, since it has ability to provide timeliness guarantees by using the *Guaranteed Time Slot* (GTS) mechanism.

In beacon-enabled mode, beacon frames are periodically sent by a central node, called PAN coordinator, to synchronize nodes that are associated with it and to describe the structure of the superframe (Figure 6). The

superframe, corresponding to the *Beacon Interval (BI)*, is defined by the time between two consecutive beacons, and includes an active period and, optionally, a following inactive period. During the inactive period, each node may enter a low-power mode to save energy resources. The active period, corresponding to the *Superframe Duration (SD)*, is divided into 16 equally-sized time slots, during which data transmission is allowed. Each active period can be further divided into a *Contention Access Period (CAP)* using slotted CSMA/CA for best-effort data delivery, and optional *Contention Free Period (CFP)*. Within the CFP, a group of time slots, called GTS, can be dedicated exclusively to a given child node. The CFP support up to 7 GTSs and each GTS may contain multiple time slots. Each GTS can transfer data either in *transmit direction*, i.e. from child to parent node (upstream flow), or *receive direction*, i.e. from parent to child node (downstream flow). Hence, the maximum numbers of child routers and end nodes are constrained as follows: $N_{router}^{MAX} + N_{end_node}^{MAX} \leq 7$.

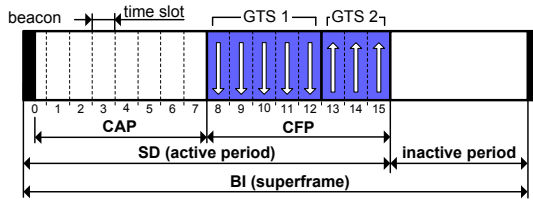


Figure 6. IEEE 802.15.4 superframe structure.

The structure of the superframe is defined by two parameters, the *Beacon Order (BO)* and the *Superframe Order (SO)*, as follows:

$$\left. \begin{aligned} BI &= aBaseSuperframeDuration \cdot 2^{BO} \\ SD &= aBaseSuperframeDuration \cdot 2^{SO} \end{aligned} \right\} \text{ for } 0 \leq SO \leq BO \leq 14$$

where $aBaseSuperframeDuration = 15.36$ ms assuming the 2.4 GHz ISM frequency band with 250 kbps data rate. The standard offers three unlicensed frequency bands: 2.4 GHz (worldwide), 915 MHz (e.g. North America) and 866 MHz (e.g. Europe). In this paper, we only consider the 2.4 GHz band with 250 kbps data rate, which is also supported by the TelosB nodes [19] using as our experimental platform.

While IEEE 802.15.4 in beacon-enabled mode supports only star-based topologies, ZigBee standard has proposed its extension to cluster-tree based topologies, where PAN Coordinator is identified as the root of the tree and forms the initial cluster. The other routers join the cluster-tree in turn by establishing themselves as the cluster-heads that subsequently start to generate the beacon frames for their clusters. Note that each cluster is active during its *SD* and inactive during the rest of its *BI*. To avoid the collisions between multiple superframe durations, the appropriate scheduling of SDs must be used (Section 3.3). For the sake of simplicity, we assume that all clusters have the same duty cycle, and whole WSN is inside one collision domain. Hence, the TDCS is given by the non-overlapping sequence of equally-sized SDs (Figure 7), and the duration of a TDCS cycle is equal to *BI*.

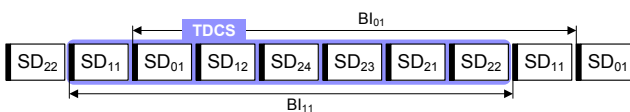


Figure 7. The worst-case TDCS related to the example in Figure 3.

6.2 Guaranteed Bandwidth of a GTS Time Slot

Each GTS time slot has a portion used for effective data transmission and a portion used by overheads (i.e. inter-frame spacing, potential acknowledgment time, and wasted time). Consecutive frames are separated by inter-frame spacing (IFS) periods. The IFS is equal to a SIFS (Short Inter-Frame Spacing) of a duration of at least 0.192 ms [6], for frame lengths smaller than or equal to $aMaxSIFSFrameSize$ (= 144 bits [6]). Otherwise, the IFS is equal to a LIFS (Long Inter Frame Spacing) of a duration of at least 0.64 ms [6], for frame lengths greater than $aMaxSIFSFrameSize$ and smaller than $aMaxPHYPacketSize$ (= 1016 bits [6]), which is the maximum size of a MAC frame, called MPDU (MAC Protocol Data Unit). In practise the applications in WSNs usually use the frames smaller than the maximum size, thus to achieve more accurate results we introduce parameter $MPDU_{max}$ representing the user define maximum size of MAC frames. In case of acknowledged transmission, the sender waits for the corresponding acknowledgment frame at most $macAckWaitDuration$ (= 0.864 ms [6]). The whole transmission, including the frame, IFS and potential acknowledgment, must be complete before the end of the GTS. Otherwise, it must wait until the next GTS. Hence, the time of the GTS can be wasted if no frame is available for transmission, or the remaining time at the end of the GTS is not enough to complete whole transmission.

We derive the expression for the guaranteed bandwidth of one time slot, which is related to the maximum effective data transmission. The maximum time required for the whole transmission of a MAC frame (MPDU) is then expressed as:

$$T_{MPDU} = MPDU_{max}/C + \Delta IFS + macAckWaitDuration \cdot \Omega$$

where $MPDU_{max}$ is the user defined maximum size of the frame, C is the data rate (we assume 250 kbps), ΔIFS is equal at least SIFS or LIFS, and $\Omega = 1$ for an acknowledged transmission or $\Omega = 0$ for an unacknowledged transmission. The maximum number of MAC frames with used defined maximum size that can be transmitted during one time slot is expressed as:

$$N_{MPDU} = \left\lfloor \frac{TS}{T_{MPDU}} \right\rfloor$$

where TS is the duration of a time slot and is equal to $SD/16$. In the remaining time, a frame smaller than $MPDU_{max}$ can be transmitted if the whole transmission can be completed before the end of the GTS. The transmission time of last frame is then expressed as:

$$T_{last_MPDU} =$$

$$TS - N_{MPDU} \cdot T_{MPDU} - \Delta IFS - macAckWaitDuration \cdot \Omega$$

Finally, assuming a full duty cycle (i.e. $SO = BO$) the guaranteed bandwidth of one GTS time slot is expressed as:

$$R_{TS}^{100\%} = \frac{N_{MPDU} \cdot MPDU_{max} + \max(T_{last_MPDU}, 0) \cdot C}{SD} \quad (32)$$

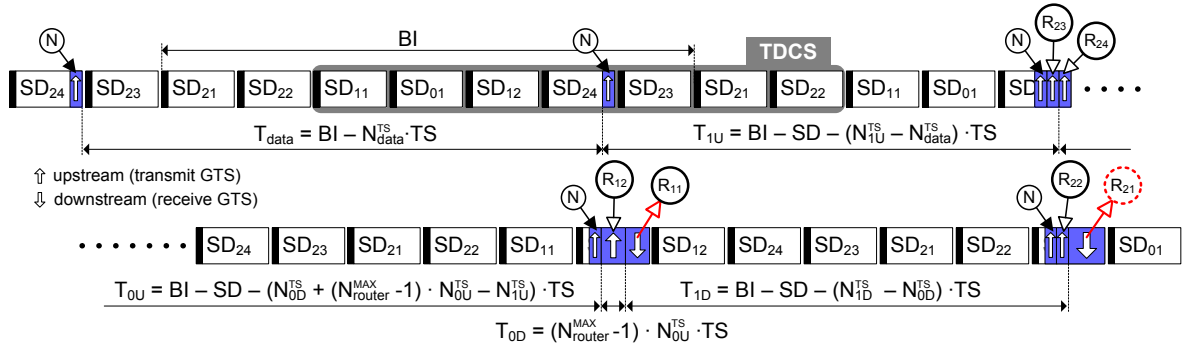


Figure 8. The worst-case service latencies for a flow along the longest path in the WSN related to the example in Figure 3.

6.3 Characterization of the Service Curve

Each parent router must reserve a GTS with enough time slots for each of its child nodes. For downstream data link, a parent router at depth i must reserve a GTS with N_{iD}^{TS} time slots in receive direction to its child router at depth $i+1$ such that the resulting link bandwidth is greater than or equal to its total input arrival rate \bar{r}_{iD} . It results that:

$$N_{iD}^{TS} = \left\lceil \frac{\bar{r}_{iD}}{R_{TS}} \right\rceil \quad (33)$$

On contrary, for upstream data link, the resulting bandwidth of GTS, guaranteed by a parent router at depth i and given by N_{iU}^{TS} time slots in transmit direction, must be greater than or equal to the total input arrival rate $\bar{r}_{(i+1)U}$ of a child node at depth $i+1$. Note that N_{data}^{TS} is number of GTS time slots guaranteed to each end-node by its parent router. Hence, a GTS with N_i^{TS} time slots provides rate-latency service $\beta_{R_i T_i}(t)$, where $R_i = N_i^{TS} \cdot R_{TS}$ is the guaranteed bandwidth and T_i is the maximum latency that a data must wait for a service.

The service latencies depend on the TDCS such that their worst-case values are achieved for the worst-case TDCS of a data flow along the longest path in a WSN. Since our methodology is based on the symmetric properties of cluster-tree topology, the same service latencies, equal to the worst-case ones, are guaranteed to all data links in given direction and depth. Let us consider the example in Figure 3, where an end-node of router R_{24} sends sensory data to the sink associated with the router R_{21} (i.e. a flow along the longest routing path). Thus, the corresponding worst-case TDCS may be given by the following sequence of superframe durations: SD₁₁, SD₀₁, SD₁₂, SD₂₄, SD₂₃, SD₂₁, SD₂₂ (Figure 8). The worst-case service latencies at each depth, except depth 0, are given by distance between the active periods of consecutive clusters on the longest routing path to the sink.

According to Figure 8, the worst-case service latency guaranteed to a flow over *upstream data link* at given depth is expressed as:

- the latency guaranteed by a router to its end node:

$$T_{data} = BI - N_{data}^{TS} \cdot TS$$
- the latency guaranteed by a router at depth i to a child router at depth $i+1$, for $\forall i, 0 < i < H$:

$$T_{iU} = BI - SD - (N_{iU}^{TS} - N_{(i+1)U}^{TS}) \cdot TS$$

- the latency guaranteed by the router at depth 0 to the child router at depth 1:

$$T_{0U} = BI - SD - (N_{0D}^{TS} + (N_{router}^{MAX} - 1) \cdot N_{0U}^{TS} - N_{1U}^{TS}) \cdot TS$$

According to Figure 8, the worst-case service latency guaranteed to a flow over *downstream data link* at given depth is expressed as:

- the latency guaranteed by a router at depth 0 to the child router at depth 1 (priority rule, Section 3.3):

$$T_{0D} = (N_{router}^{MAX} - 1) \cdot N_{0U}^{TS} \cdot TS$$

- the latency guaranteed by a router at depth i to the child router at depth $i+1$, for $\forall i, 0 < i < H_{sink}$:

$$T_{iD} = BI - SD - (N_{iD}^{TS} - N_{(i+1)D}^{TS}) \cdot TS$$

6.4 IEEE 802.15.4/ZigBee Cluster-Tree WSN Setup

For our experimental scenario, we consider a simple cluster-tree WSN corresponding to the configuration where $H = 2$, $N_{end_node}^{MAX} = 1$, $N_{router}^{MAX} = 2$. For the sake of simplicity, only end-nodes are equipped with sensing capability (i.e. $S = 0$) and generate data flows bounded by the arrival curve $\alpha_{data}(t)$. We assume a minimum possible value of SO (e.g. $SO = 4$, imposed by some technological limitation of our experimental platforms, namely due to the non-preemptive behaviour of the TinyOS [20] operating system. According to Eq. (29), the total number of routers is equal to 7. Hence, BO must be set such that at least 7 SDs with $SO = 4$ can fit inside the BI without overlapping. In general, we obtain:

$$BI \geq \Sigma(N_{router}^{MAX}, H) \cdot SD \Leftrightarrow BO_{min} = \lceil \log_2(\Sigma(N_{router}^{MAX}, H) \cdot 2^{SO}) \rceil$$

As a result for $SO = 4$, the minimum BO is equal to 7, such that a maximum of $2^7/2^4 = 8$ SDs can fit in one BI . The maximum duty cycle of each cluster is then equal to $(1/8) = 12.5\%$. Note that to maximize the lifetime of a WSN, the lowest duty cycles must be chosen. The IEEE 802.15.4 can provide very low duty cycles, up to 0.1%. As a result, the inactive period of BI is extended, and the nodes may stay in low-power mode longer to save energy resources. On the other hand, low duty cycles enlarge end-to-end delays. Hence, long lifetime is in contrast to the fast timing response of a WSN, so a trade-off must be found. In our example with $SO = 4$, we can get the duty cycles: 12.5% ($BO = 7$), 6.25% ($BO = 8$), 3.125% ($BO = 9$), and so on.

According to [6], the minimum CAP (i.e. $aMinCAPLength$ parameter) is equal to 7.04 ms, assuming the 2.4 GHz ISM band, which corresponds to 1 time slot

with $SO = 4$. The remaining slots can be allocated for GTSSs. Hence, the maximum CFP length is equal to $L_{CFP} = 15$ time slots. With this constraint, a router cannot reserve more than L_{CFP} time slots for 7 GTSSs maximum, i.e. for its $N_{end_node}^{MAX}$ end-nodes and N_{router}^{MAX} child routers. Assuming that each end-node requires allocation of a GTSS with N_{data}^{TS} time slots (i.e. $r_{data} \leq N_{data}^{TS} \cdot R_{TS}$) from its parent router. Then, each child router can allocate a GTSS with the maximum number of time slots equal to:

$$\lfloor (L_{CFP} - N_{data}^{TS} \cdot N_{end_node}^{MAX}) / N_{router}^{MAX} \rfloor$$

According to Eq. (28), the arrival rate r_{data} must be limited in order not to exceed the maximum bandwidth that a parent router can reserve. Obviously, due to the cumulative flow effect, the maximum bandwidth will be required by the sink router. Hence, the corresponding link bandwidth guaranteed by the parent router at depth $H_{sink} - 1$ to the sink router at depth H_{sink} is equal to:

$$R_{(H_{sink}-1)} = \left\lfloor \frac{L_{CFP} - N_{data}^{TS} \cdot N_{end_node}^{MAX}}{N_{router}^{MAX}} \right\rfloor \cdot R_{TS} \quad (34)$$

As a result applying Eq. (28), we obtain the maximum arrival rate of the sensory data flow as:

$$r_{data}^{MAX} = \left(\frac{\left\lfloor \frac{L_{CFP} - N_{data}^{TS} \cdot N_{end_node}^{MAX}}{N_{router}^{MAX}} \right\rfloor \cdot R_{TS}}{\left(\sum_{j=0}^{(H_{sink}-1)} (N_{router}^{MAX})^{H-j} \right) \cdot (N_{end_node}^{MAX} + S_{ij})} \right) \quad (35)$$

Note that the aforementioned expressions are valid for $\forall H_{sink}, 1 \leq H_{sink} \leq H$. The expressions for $H_{sink} = 0$ have been already derived in [5]. As a result the average arrival rate r_{data} of sensory data flow must be lower than r_{data}^{MAX} in any case. The value of burst b_{data} is selected according to the burstiness of sensory data.

7. EXPERIMENTAL EVALUATION

In this section, we compare the analytical results based on Network Calculus that we proposed in this paper with the experimental results obtained through the use of IEEE 802.15.4/ZigBee technologies. The analytical results are computed using a MATLAB model [18], and the experimental results are obtained using a real test-bed based on the TelosB motes [19].

7.1 Experimental Setup

The experimental test-bed (illustrated in Figure 9) consists of 14 TelosB motes running the TinyOS [20] operating system with our open source implementation of the IEEE 802.15.4/ZigBee protocol stack [21]. The TelosB is a battery powered wireless module with integrated sensors, IEEE 802.15.4 compliant radio, antenna, 16-bit RISC microcontroller, and programming capability via USB. To capture the traffic, we have used the packet sniffer from Texas Instruments (formerly Chipcon) [22] that provides a raw list of the transmitted packets, and the Daintree Sensor Network Analyzer (SNA) [23] that provides additional functionalities, such as the graphical topology of the network. Note that the experimental deployment can span over a wider region, in practise. Each router, except the root, must hear its end nodes, child routers and parent router.

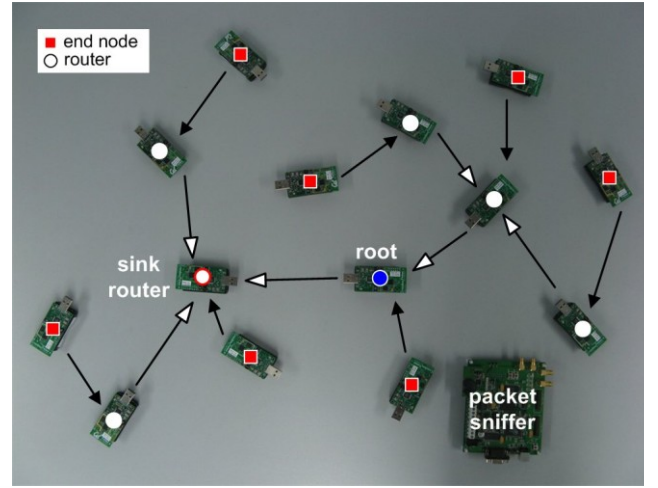


Figure 9. The test-bed deployment for $H_{sink}=1$.

The analytical model [18] was developed in MATLAB, and can run in Command Line Interface (CLI) mode or Graphical User Interface (GUI) mode. On the left side of the GUI in Figure 10, the network and sensory data flow parameters are entered. After the computation, the results and optionally several charts are shown on the right side. The values in Figure 10 correspond to the under mentioned network setting and the results from Section 7.2, namely the worst-case end-to-end delays for $H_{sink} = 0$.

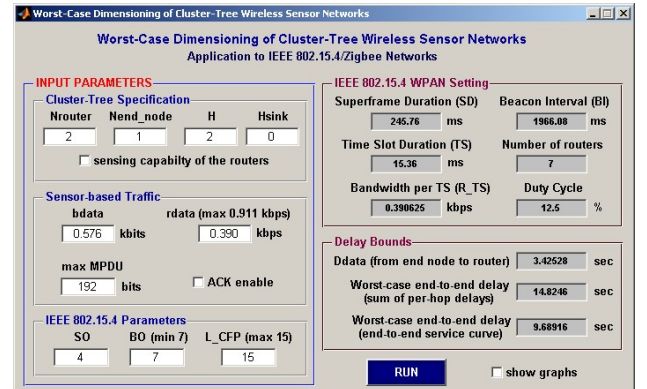


Figure 10. The GUI of the MATLAB analytical model.

We consider a cluster-tree WSN with the configuration mentioned in Section 6.4. We configured the application running on the sensor nodes to generate 3 bytes at the data payload. Hence, the maximum size of the MAC frame is equal to $MPDU_{max} = 192$ bits (i.e. MAC Header = 88 bits, FCS = 16 bits, Network Header = 64 bits, and Data Payload = 24 bits). Note that all devices in WSN have unique 16-bit short addresses allocated by the PAN Coordinator during the association process.

TinyOS 1.x flushes the reception buffer of the radio transceiver after processing the first arriving frame. Thus, the frames that arrive during the processing time of the first frame are discarded. This problem has been already reported and fixed in TinyOS 2.x. Since our implementation of IEEE 802.15.4/ZigBee protocol stack was built over TinyOS 1.x, we overcame the aforementioned problem by setting the inter-frame spacing (IFS) time (i.e. time between two consecutive frames) such that no frame arrives during the frame processing times. IFS has been set to 3.07 ms.

According to Eq. (32), the bandwidth guaranteed by one time slot for $SO = 4$ is equal to 3.125 kbps with 100% duty

cycle. Hence, in our experimental scenario with a 12.5 % duty cycle (i.e. $BO = BO_{min} = 7$), the guaranteed bandwidth of one time slot is equal to $R_{TS} = 3.125 \cdot 0.125 = 0.3906$ kbps. Let us assume $N_{data}^{TS} = 1$. Then according to Eq. (35), we obtain the maximum arrival rates of the sensory data flow as follows:

- $r_{data}^{MAX} = 456$ bps for $H_{sink} = 2$
- $r_{data}^{MAX} = 684$ bps for $H_{sink} = 1$
- $r_{data}^{MAX} = 911$ bps for $H_{sink} = 0$ (root)

As a result of $r_{data} \leq \min(r_{data}^{MAX})$ and $r_{data} \leq R_{TS}$, we consider an average arrival rate equal to $r_{data} = 390$ bps, which corresponds to 4 frames (192 bits each) generated during one Beacon Interval ($BI = 1.96608$ sec). We assume that the burst tolerance is equal to $b_{data} = 576$ bits, which corresponds to 3 frames generated at once. Hence, each sensory data flow is bounded by arrival curve $\alpha_{data}(t) = 576 + 390 \cdot t$. Note that Network Calculus based analytical model is bit oriented, while the experimental test-bed is frame oriented. Let us assume that during the first BI is generated the maximum number of frames that can be generated during any BI, i.e. 7 (3+4). Then at the beginning of next BI no frame can be generate, and at most 4 frames can be generated during this BI. The frames can be generated as constant bitrate (CBR) or variable bitrate (VBR) traffic upper bounded by the arrival curve $\alpha_{data}(t)$ (Figure 11).

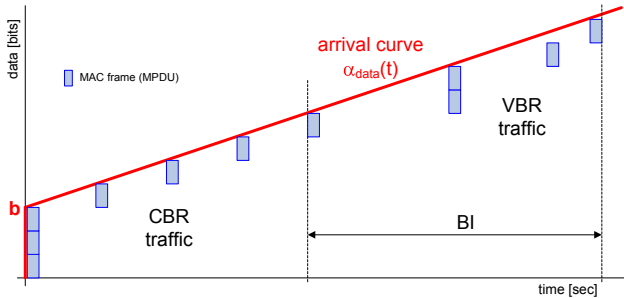


Figure 11. The sensory traffic generation.

Finally, let us summarize the complete network setting:

- $N_{router}^{MAX} = 2$
- $N_{end_node}^{MAX} = 1$
- $H = 2$
- $SO = 4$ ($SD = 245.76$ ms)
- $BO = 7$ ($BI = 1966.08$ ms)
- Duty Cycle = 12.5 %
- $MPDU_{max} = 192$ bits
- $r_{data} = 390$ bps
- $b_{data} = 576$ bits
- $IFS = 3.07$ ms
- $L_{CFP} = 15$
- $S = 0$

We assume the worst-case TDCS of a flow along the longest routing path from router R_{24} to the sink (Figure 3) given by the following sequence of superframe durations: SD_{11} , SD_{01} , SD_{12} , SD_{24} , SD_{23} , SD_{21} , SD_{22} . Note that we consider only unacknowledged transmissions and all nodes inside one collision domain.

7.2 Experimental vs. Theoretical Results

7.2.1 Buffer Requirements

Figure 12 presents the theoretical worst-case buffer requirement of the routers at given depth as a function of the sink position. It can be observe that end-nodes have the smallest buffer requirement as they are the leaves of the tree, and that the buffer requirement grows in direction of the sink router. Since the sink can be associate with any router in a WSN and in order to avoid buffer overflow, all routers at depth i should allocate a buffer of capacity equal at least to the maximum buffer requirement at given depth

i (e.g. all router at depth 0 allocate a buffer of capacity equal to 15.995 kbits), which effectively demonstrates how these analytical results can be used by a system designer.

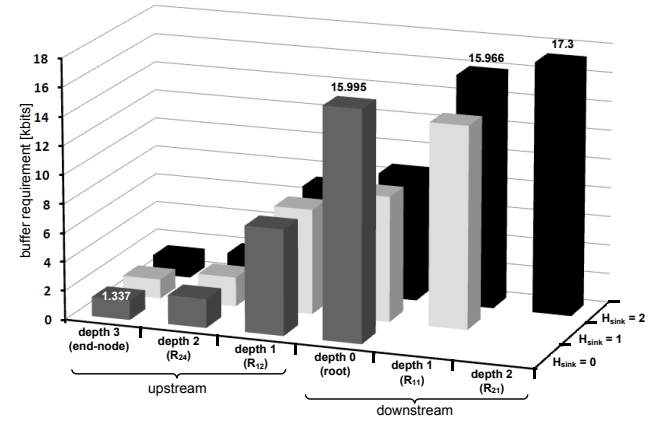


Figure 12. The worst-case buffer requirements per router as a function of the depth and sink position.

Figure 13 shows the theoretical worst-case buffer requirements compared with the maximum values obtained through real experimentation, for $H_{sink} = 2$.

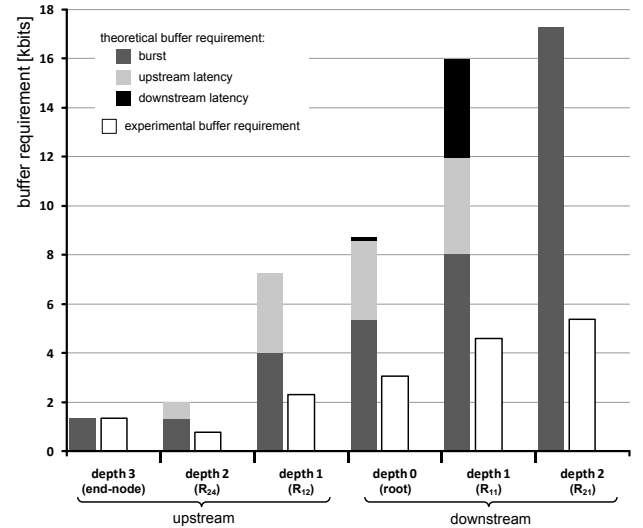


Figure 13. The theoretical vs. experimental buffer requirements.

First, the theoretical buffer requirements are divided into three portions according their origin, as we have shown in Section 5.1.2. Observe that the cumulative effect of the burst is more important than the cumulative effect of the service latencies. The effect of the service latencies may be more important for other setting of b_{data} and r_{data} . So, the different setting of the sensory arrival curve affects the buffer requirements. The minor effect of the upstream service latency at depth 0 is given by the priority rules (Section 3.3), such that the data arriving during the transmit GTS (i.e. upstream flow) are stored in the root until the receive GTS (i.e. downstream flow), at the end of the same SD, is active and data is dispatched (Figure 8).

The next observation confirms that the theoretical values upper bound the experimental values. The pessimism of the theoretical bounds is justified by the fact that the Network Calculus analytical model is based on a continuous approach (arrival and service curves are continuous) in contrast to the real stepwise behaviour of flows and services in the test-bed. In practice, the data is actually transmitted only during its GTS, while in the analytical model we consider a continuous data flow

during the whole BI , since it represents the average rate and not the instantaneous rate. Figure 14 illustrates the problem and shows the arrival and service curves of a data flow sent by an end-node to its parent router. The burst of the outgoing data flow b_{data}^* (Eq. (11)) is equal to Q_{max}^{TH} , in case of the analytical model, or Q_{max}^{EXP} , in the experimental case. Due to the cumulative flow effect, the differences between theoretical (Q_{max}^{TH}) and experimental (Q_{max}^{EXP}) values of buffer requirement grow with depth. The rate-latency service curve used in our analysis results from a trade-off between computing complexity and pessimism.

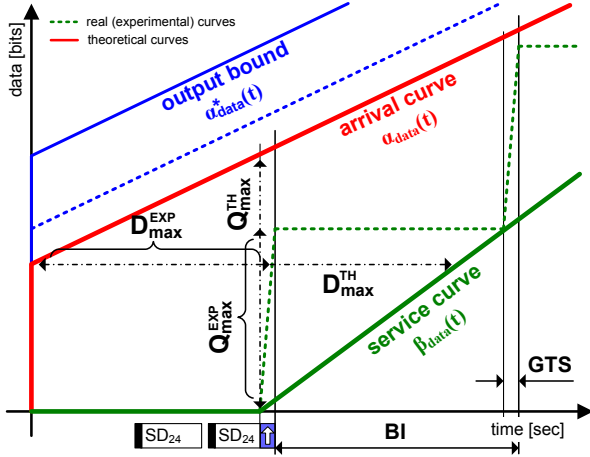


Figure 14. Theoretical vs. experimental data traffic (related to Figure 2).

The numerical values of theoretical worst-case as well as experimental maximum buffer requirements are summarized in Table 1. The bandwidth requirements given by Eq. (28) and the corresponding number of time slots are also presented. In the Tables 1 and 2, U means upstream router at depth i or upstream link to a router at depth i , and D means downstream router or downstream link from a router at depth i .

TABLE 1
BUFFER REQUIREMENTS: THEORETICAL VS. EXPERIMENTAL RESULTS

	depth	theoretical results (worst-case values)			experimental results (maximum values)
		R_i [kbps]	N_i^{TS}	Q_i [kbits]	
$H_{sink} = 0$ (root)	0 U	1.7	3	15.995	5.376
	1 U	0.39	1	7.329	2.304
	2 U	—	—	2.008	0.768
$H_{sink} = 1$	0 D	1.56	4	8.667	3.072
	U	1.17	3	—	—
	1 D	—	—	14.02	5.376
	U	0.39	1	7.257	2.304
	2 U	—	—	2.008	0.768
$H_{sink} = 2$	0 D	1.56	4	8.667	3.072
	U	1.17	3	—	—
	1 D	2.34	6	15.966	4.608
	U	0.39	1	7.257	2.304
	2 D	—	—	17.3	5.376
	U	—	—	2.008	0.768
	end-node	0.39	1	1.337	1.344

7.2.2 Delay Bounds

In Figure 15, we compare the worst-case, maximum and average values of per-hop delays bound in each router, and the end-to-end delay bounds for $H_{sink} = 2$. A first observation confirms that theoretical values upper bound the experimental values. The difference in theoretical worst-case (D_{max}^{TH}) and experimental maximum (D_{max}^{EXP})

delays (Figure 14) is given by the aforementioned continuous and stepwise behaviours of the analytical model and test-bed, respectively. The experimental delays comprise mainly the service latencies (Figure 14) decreasing in the direction of the sink (Figure 8). Hence, the maximum per-hop delays also decrease in the direction of the sink as you can observe in Figure 15. The low downstream delay at depth 0 results from priority rule (Section 3.3). The end-to-end delays bounds are quite high, even though the b_{data} and r_{data} are low. This is mainly due to high value of $SO = 4$ (i.e. $BI = 1.966$ sec). Hence, the end-to-end delay bounds can be reduced using lower values of SO or higher bandwidth guarantees, using lower IFS, for example.

Observe also that the worst-case end-to-end delay obtained by the per-flow approach offers less pessimism than the delay from the per-hop approach.

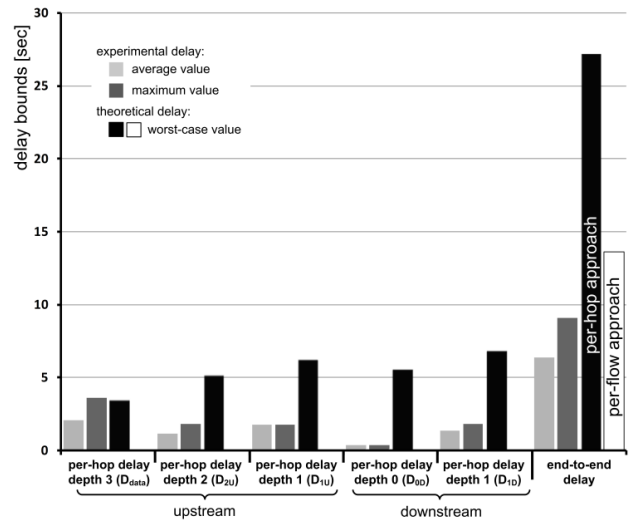


Figure 15. The theoretical vs. experimental delay bounds.

Table 2 presents the worst-case, maximum and average numerical values of per-hop and per-flow delay bounds, and the end-to-end delays for given sink position.

TABLE 2
DELAY BOUNDS: THEORETICAL VS. EXPERIMENTAL RESULTS

	depth	theoretical results (worst-case values)	experimental results	
		D_i [sec]	maximum	average
$H_{sink} = 0$ (root)	1 U	6.257	1.764	1.308
	2 U	5.143	1.812	1.602
	D_{e2e}	14.82/ 9.69	7.154	4.952
$H_{sink} = 1$	0 D	5.547	0.104	0.099
	1 U	6.195	1.76	1.728
	2 U	5.143	1.809	1.602
	D_{e2e}	20.31/ 10.53	7.251	5.471
	0 D	5.547	0.104	0.099
$H_{sink} = 2$	1 D	6.814	1.812	1.321
	U	6.195	1.766	1.728
	2 U	5.143	1.814	1.135
	D_{e2e}	27.13/ 13.65	9.074	6.325
	end-node (D_{data})	3.425	3.578	2.042

Note that the average values were computed from the set of 15 measurements with 1155 frames. The theoretical worst-case end-to-end delays are obtained as the sum of per-hop delays using Eq. (31) (first term), or by per-flow approach (Section 5.2.2) using recursively aggregate scheduling theorem (10) which results in the family of service curves as a function of $\theta \geq 0$. In our analysis we assume $\theta = T + (b_2/R)$ as a trade-off between computation

complexity and optimality. The optimal service curve, which offers the shortest delay, is given by $\beta_1(t, \theta) = b_1$, and it is the aim of our future work.

8. CONCLUSIONS AND FUTURE WORK

In this paper, we tackled the worst-case dimensioning of cluster-tree wireless sensor networks (WSN) assuming that the data sink can be mobile, i.e. can be associated to any router in the sensor network. We provided a system model, an analytical methodology and a software tool that enables system designers to dimension and analyze these networks. In this way, it is possible to minimize the routers' buffers size to guarantee there will be no overflow and to minimize each cluster's duty cycle (maximizing nodes' lifetime) still satisfying that messages' deadlines are met.

Importantly, we showed how it is possible to instantiate our generic methodology in IEEE 802.15.4/ZigBee, which are the leading technologies for WSNs. We also developed a 7 clusters test-bed based on Commercial-Off-The-Shelf technologies, namely TelosB motes [19] running our open-ZB protocol stack [21] over TinyOS [20]. This test-bed enabled to assess the pessimism of our worst-case theoretical results (buffer requirements and message end-to-end delays), by comparing these to the maximum and average values measured in the experiments.

Ongoing and future work include improving the current methodology to encompass clusters operating at different duty-cycles and to provide a model that enables real-time control actions, i.e. the sink assuming also the role of controlling sensor/actuator nodes.

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